# *Abstract*


The main goal of this thesis is studying some geometrical aspects in univalent and multivalent function theory. It is the studying of differential sandwich theorems of analytic functions defined by generalized integral operator, here, we obtain some applications of first order differential subordination and superordination results involving a generalized integral operator for certain normalized analytic functions, like, let $q(z)$ be univalent in the open unit disk $U$ with $q(0) = 1, \beta \in \mathbb{C}^*, \gamma > 0$ and suppose that

$$Re\left(1 + \frac{zq''(z)}{q'(z)}\right) > max\left(0, -Re\left(\frac{\gamma(\alpha\delta + p)}{\beta\gamma}\right)\right),$$

if $f \in \mathcal{A}(p)$ satisfies the subordination

$$(1-\beta)\left(\frac{I_p^{m+2}f(z)}{z^p}\right)^\sigma + \beta\left(\frac{I_p^{m+2}f(z)}{z^p}\right)^\sigma \frac{I_p^{m+1}f(z)}{I_p^{m+2}f(z)} \prec q(z) + \frac{\beta\gamma}{\gamma(\alpha\delta + p)} zq'(z),$$

then $\left(\frac{I_p^{m+2}f(z)}{z^p}\right)^\sigma \prec q(z)$, and $q(z)$ is the best dominant.

We have also studied subordination properties of univalent functions. We derive some subordination properties, like, if the function $q$ be univalent in the open unit disk $U$, $q(z) \neq 0$ and $zq'(z)\theta(q(z)) \neq 0$ is starlike in $U$. If $f \in \Sigma_\alpha^+$ satisfies the subordination

$$2 - \frac{zf'''(z)}{f''(z)} + \frac{z^2 f''(z)}{zf'(z) - f(z)} \prec \frac{-zq'(z)}{\delta q(z)}. \text{ Then } \left[\frac{z^2 f''(z)}{zf'(z) - f(z)}\right] \prec q(z),$$

and $q(z)$ is the best dominant. Also we have given differential subordination of classes of univalent functions, we obtain main results of these classes related with differential subordination, like, if the function $q$ be convex univalent in the open unit disk $U$, $q'(z) \neq 0$ and assume that

$$Re\left(1 + \frac{zq''(z)}{q'(z)} + \frac{1}{t}\right) > 0,$$

where $t \in \mathbb{C} / \{0\}$. If $f \in W(\lambda)$ satisfies the subordination,

$$\left[\frac{zf''(z) - f'(z)}{f'(z)}\right]^\delta + t\delta\left[\frac{zf''(z) - f'(z)}{f'(z)}\right]^\delta \left[\frac{z^2 f'''(z)}{zf''(z) - f'(z)} - \frac{zf''(z)}{f'(z)}\right] \prec q(z) + tzq'(z),$$

then

$$\left[\frac{zf''(z) - f'(z)}{f'(z)}\right]^\delta \prec q(z), z \in U, f'(z) \neq 0, \delta \in U/\{0\},$$

and $q$ is the best dominant.

We have discussed and studied the subclass $FW_{\mathcal{H},m}(\gamma,\alpha,\beta,\sigma,\rho,\delta)$ of harmonic univalent functions defined by a generalized integral operator of the form $f = h + \bar{g}$, where

$$h(z) = z - \sum_{n=2}^{\infty} a_n z^n\,,\;\; g(z) = -\sum_{n=1}^{\infty} b_n z^n,\;\; (a_n \geq 0, b_n \geq 0, |b_1| < 1),$$

satisfying the condition :

$$Re\left(\frac{z(I^m f(z))'' + (I^m f(z))'}{(I^m f(z))' + \gamma z(I^m f(z))''}\right) > \beta\left|\frac{z(I^m f(z))'' + (I^m f(z))'}{(I^m f(z))' + \gamma z(I^m f(z))''} - 1\right| + \alpha,$$

where $0 \leq \alpha < 1$ , $\beta \geq 0$ , $0 \leq \gamma < 1$ and $z \in U$.

We obtain some interesting properties for this class, like, coefficient bounds, extreme points, convex combination, distortion bounds and Hadamard product.

We have also studied and introduced a new class $\Sigma(\alpha,\gamma)$ of meromorphic univalent functions of the form :

$$f(z) = z^{-1} + \sum_{k=1}^{\infty} a_k z^k\,, (a_k \geq 0, k \in \mathbb{N} = \{1,2,3,\dots\}), z \in U^* = \{z \in \mathbb{C}: 0 < |z| < 1\},$$

and satisfying the condition :

$$\left|\frac{z(z^2 f'(z))' - z^2 f'(z)}{\gamma - z^2 f'(z)}\right| < \alpha \;\text{, where}\quad (0 < \alpha \leq 1, \gamma > 0).$$

We obtain some geometric properties, like, coefficient inequality, convex set, arithmetic mean, distortion bounds, partial sums, neighborhood property, radii of starlikeness and convexity and Hadamard product .

Also, we have discussed generalization of a certain subclass $\mathcal{F}_j(n,m.p,q,\alpha,r,\beta)$ of p-valent analytic functions with negative coefficients defined by a differential operator of the form :

$$f(z) = z^p - \sum_{k=j+p}^{\infty} a_k z^k\;,\quad (\,a_k \geq 0; p,j \in \mathbb{N}\,),$$

satisfying the condition :

$$Re\left(\frac{(p-q-r)D_p^{n+m} f^{(q+r)}(z)}{D_p^n f^{(q+r)}(z)}\right) > \beta\left|\frac{(p-q-r)D_p^{n+m} f^{(q+r)}(z)}{D_p^n f^{(q+r)}(z)} - 1\right| + \alpha\,,$$

$$(p,m \in N; q,r,n \in N_0; 0 \leq \beta < 1).$$

For some $\alpha$ $(0 < \alpha < p-q-r, p > q-r)$ and for all $z \in U$.

We obtain some results, like, coefficient estimates, distortion bound, radii of starlikeness, convexity and close-to-convexity, closure theorem and applications of fractional calculus operators.