ABSTRACT

This thesis deals with some new properties and extensions on fuzzy Hilbert spaces. The study also introduces a theorem that is equivalent to the definition of the fuzzy pre-Hilbert space (the real part).

After that we have obtain two decomposition theorems from a fuzzy pre-Hilbert space into a ascending family of fuzzy pre-Hilbert spaces, then we study fuzzy orthonormal set and some theorems to which they relate , we have provide some of properties and definitions adjoint of the operator on fuzzy Hilbert spaces and invested some important general properties of adjoint fuzzy linear operators on fuzzy Hilbert spaces.

We have introduced the set $F(X)$ which is the set of all bonded fuzzy functions sets and we introduced the definition of a two-fuzzy metric space and we have obtained some new properties of these sets by studying the open and closed balls, as well as study property fuzzy convergence and fuzzy closure set in the two-fuzzy metric space.

The study sheds some light on the two-fuzzy normed space and two-fuzzy pre-Hilbert space concentrating on some of their properties like convergence, continuity and the orthogonality in two-fuzzy pre-Hilbert space in order to study the relationship between these spaces.