Abstract

The main aim of this work is to create a new type of proper G-space namely, "Generalized Proper G-space" and study of it is properties.

We introduce a new definition for a generalized proper function which is considered as the basis of our main definition. Also, we study the properties of this function.

Throughout this work, some important and new concepts have been illustrated including a generalized convergence of nets, generalized compactly generalized closed spaces and generalized Cartan G-space and clarified their properties.

Finally, we have introduced the definitions of generalized limit sets and use it to characterize certain types of generalized G-space.

Also, this study proves that:

- (i) Let \boldsymbol{X} be a T_2 -space , then:
- (\bullet) A subset A of a topological space X is cgc-set if and only if it is g-closed set .
- (•) A cgc-space and gcgc-space are equivalent .
- (ii) Let $f: X \to Y$ be a g-proper function , then the restriction of f on a closed subset F of a space X is g-proper function .
- (iii) Let $f: X \to Y$ and $h: Y \to Z$ be a g-continuous functions, then:
- (\bullet) If hof is g-proper function and f is onto , where X , Y are T_1 -spaces , then h is g-proper.
- (•) If hof g-proper and h is gl-continuous, one to one , then f is g-proper.
- (iv) If $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$ be two g-proper functions, Then $f_1 \times f_2$ is g-proper function .

- (v) Let $f:X\to Y$ be a g-proper function , where X g-compact and X , Y are T_1 spaces , then f is g-compact .
- (vi) If X and G are g-compact, T_1 -space and X be a g-proper G-space, then the orbit space X/G is gT_2 -space. Moreover If G is gT_2 -space, then X is also .
- **(vii)** Let (f,h) be a morphism from G_1 -space X_1 into G_2 -space X_2 such that $f\colon G_1\to G_2$ is onto and $h\colon X_1\to X_2$ is homeomorphism, where X_i are T_1 -spaces if X_1 is g-proper G_1 -space, then X_2 is g-proper G_2 -space.
- (iix) Let X be a G-space, then:
- (•) If K_1 , K_2 are g-compact subsets of X then $((K_1, K_2))$ is g-closed subset of G and $((K_1, K_2))$ is g-compact when K_1 and K_2 are relatively g-thin .
- (•) If $\Lambda^g(x) = \emptyset$ for each $x \in X$, then the orbit G.x is not g-compact.
- (•) Let X and G are g-compact T_1 -spaces , then X is g-proper G-space if and only if $J^g(x)=\emptyset$ for each $x\in X$.
- (ix) Let X be a gCG-space, then $\Lambda^g(x) = \emptyset$ for each $x \in X$.
- (x) Let (I_G,h) be a morphism from G-space X_1 into a G-space X_2 such that $h\colon X_1\to X_2$ be a gl-continuous and g-homeomorphism. If X_1 is $\mathsf{gC} G$ -space, then so is X_2 .